The Works of

Archimedes of Syracuse

287 – 212 BC

While historians of mathematics generally consider Archimedes to have been one of the greatest mathematicians of all time, in his own time he was better known for his practical achievements:

> He invented the compound pulley and the Archimedean screw – still in use – for raising water; he understood the power of leverage … and he devised a variety of fearsome engines of war for the defence of his beleaguered city.[1]

"We have more of the writings of Archimedes than of any other great mathematician of antiquity."[2] In *On the Sphere and Cylinder*, Archimedes proved, among other things, that the surface area of a sphere is four times that of the largest circle it contains. Perhaps his favourite theorem, which he had inscribed upon his tombstone, was the double result that the volume and surface area of a sphere are two-thirds of the volume and surface area of the circumscribed cylinder.

In *On Spirals*, Archimedes was interested in various areas, lengths and tangents associated with what are now called 'Archimedean spirals'. One of the more important results shows that a certain straight line associated with a tangent to his spiral has the same length as the circumference of a particular circle.[3] Archimedean spirals can also be used to trisect the angle.

In *The Measurement of the Circle*, Archimedes showed that the area of a circle of radius R and circumference C is equal to the area of a right-angled triangle, where the sides containing the right-angle are of lengths R and C. Combining this result with that from *On Spirals* mentioned above, he effectively reduced the problem of squaring the circle to that of finding

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[1] Stuart Hollingdale, *Makers of Mathematics* (London: Penguin, 1994), p. 85.
[2] **SB** editors' introduction to 4.A, p. 139.
[3] For details, see **SB** 4.A7(g), p. 162-3.

a particular tangent to his spiral. By considering polygons (of 96 sides) inscribed and circumscribed about a circle Archimedes also showed, in modern terms, that $3\,^{10}/_{71} < \pi < 3\,^{1}/_{7}$.

Many of Archimedes' results, including some of the above, were reached using the method *of exhaustion*. For our purposes here, let it suffice to say that proofs using this method have a *reductio ad absurdum* argument structure and involve inscribed and/or circumscribed figures which – in a technical sense – 'exhaust' the figure they inscribe/circumscribe.[4]

However, constructing a *reductio* argument requires that one have an idea of one's conclusion before beginning the proof, for one must assume (for the sake of argument) that this conclusion is false and from this deduce a contradiction. This suggested that Archimedes had some method of analysis that didn't appear in his work.[5] This proved to be right. In 1906, J.L. Heiberg rediscovered *The Method*, which explains the mechanical method of analysis by which Archimedes came upon (some of?) the theorems that he elsewhere proves by the *method of exhaustion*. In the letter that prefaces this work, Archimedes wrote that he hoped the method described would help others to discover theorems that had not occurred to him.

Archimedes wrote several books in addition to these. *The Sand-Reckoner* develops a system for expressing large numbers, which was not easy within the standard Greek number system (the Attic system). Other works relate to volumes and areas of certain figures (*Quadrature of the Parabola* and *On Conoids and Spheroids*), and to mechanics (*Plane Equilibriums* and *On Floating Bodies*). We also know that he wrote several works that are now lost, including one, *On Sphere-Making*, which explained the construction of his Orrery.

Despite this glittering mathematical career, we must conclude that the mathematical (as opposed to mechanical) works of Archimedes were of only limited importance to Greek mathematicians. At least, we must conclude this if by 'was important to Greek

[4] For more detail see Stuart Hollingdale, *Makers of Mathematics*, pp. 30-2.
[5] See John Wallis (1685) in **SB** 4.B3, p. 177.

mathematicians' we mean 'strongly influenced the form and content of later Greek mathematical work'. Although later Greek mathematicians referred to Archimedes' works and used his results, his methods were not taken up.[6] This may have been for the simple reason that Archimedes' contemporaries and immediate successors found these methods too difficult, for as Plutarch wrote: "no amount of mental effort of his own would enable a man to hit upon the proof of one of Archimedes' theorems."[7]

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[6] Since I can find little evidence of my own for this claim, I trust it to three authorities: (i) Dr. Roger Webster, course tutor; (ii) MA290 *Unit 4: The Greek Study of Curves*, pp. 18-9; and (iii) J.J. O'Connor and E.F. Robertson, "Archimedes of Syracuse" *The MacTutor History of Mathematics Archive* <http://www-history.mcs.st-and.ac.uk/history/References/Archimedes.html> Accessed 28th March 2003.

[7] Plutarch (c. AD 46-129), **SB** 4.B1, p. 175.